Biomathematics Innovation Group
Modeling Biological Phenomena

Potential Impacts of Basic Reproduction Number and Rewiring Probability on the COVID-19 Epidemic

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Abstract

The basic reproduction number (R_0) of a disease can be thought of as the number of cases that one case will directly generate if the rest of the population is susceptible to infection. The R_0 of COVID-19 has recently become clearer; toward the beginning of the outbreak, the Imperial College Group estimated it to be somewhere between 1.5 and 3.5 (Imai *et al.* 2020), but they have now assumed a value of 2.4 with a larger range between 2.0 and 2.6 (Ferguson *et al.* 2020). Using the Watts-Strogatz small-world model, I examined the effects of different R_0 values and rewiring probabilities on the maximum number of individuals infected at a time, the length of the epidemic, the total number of individuals infected, and the day of peak infectious individuals. The rewiring probability has to do with connectedness of individuals in a population; our society is currently attempting to lower connectedness through efforts of social distancing. Results indicate a critical point at rewire.p = .05 where there is a significant change in how R_0 impacts the epidemic measures. When the rewiring probability lies between .05 and .1, there is another critical point around R_0 = 2 where epidemic measures are substantially worsened. Past R_0 of 2.5 and rewire.p ~ 0, the impacts of the connectedness of individuals on epidemic measures are less substantial, which raises concerns about the effectiveness of social distancing on COVID-19.

Objectives

I began working on this project when the COVID-19 outbreak was first identified in Wuhan, China. With little known about the virus at the time other than similarities to the SARS outbreak in 2002, I decided to take a theoretical approach to the issue. Studies were quickly generated about the reproduction number of COVID-19 based on its spread. As the virus progressed, values became more clear. The goal of my research was to examine the effects of basic reproduction number and rewiring parameter on the total individuals infected, the maximum infectious individuals at a time, the length of the epidemic, and the timing of peak infectious individuals. The impact of rewiring parameter is even more relevant now with current social distancing measures. Rewiring parameter is directly related to the connectedness of a population. I looked to see how society's attitude toward the issue (whether or not people take social distancing seriously) will impact the epidemic. I also questioned the interaction between connectedness and basic reproduction number and how that relationship has an effect on epidemic measures.

Acknowledgements

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The Model

The Watts-strogatz model of a small world network.

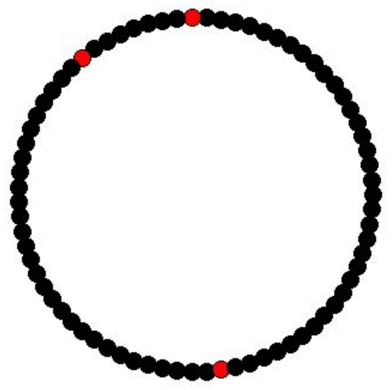

Figure 1. Small world network of 75 vertices; rewire.p = 0.0

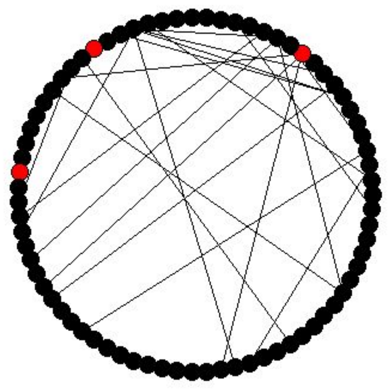

Figure 2. Small world network of 75 vertices; rewire.p = 0.05

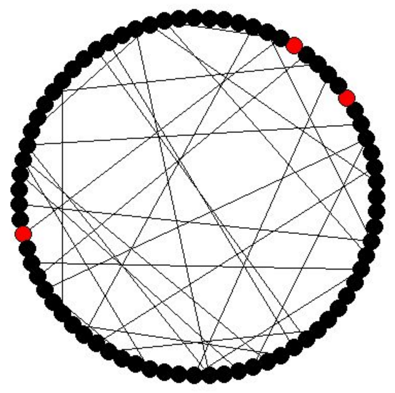

Figure 3. Small world network of 75 vertices; rewire.p = 0.1

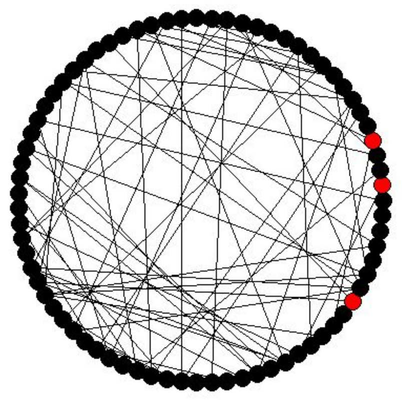

Figure 4. Small world network of 75 vertices; rewire.p = 0.25

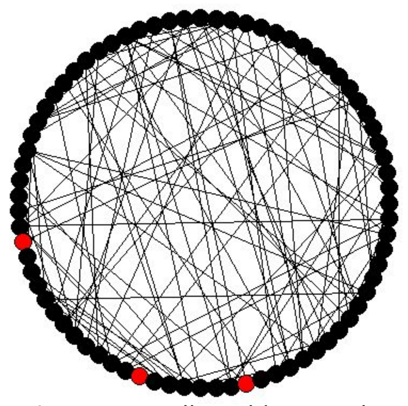

Figure 5. Small world network of 75 vertices; rewire.p = 0.5

Here is a smaller visualization of the network with only 75 nodes around a ring and three initially infectious individuals. In Figure 1, each node is connected to its two neighbors on either side. However, as the probability of rewiring increases (Figures 2 - 5), each connecting edge is disconnected from its node and reconnected to a random node in the ring with the probability of *rewire.p*; this creates shortcuts in the network (*Nykamp*). The basic reproduction number of a disease (R_0) can be thought of as the number of cases one case will generate if the rest of the population is susceptible to infection. As R_0 increases, infectious vertices in the network will each spread the disease to more individuals.

My model is a network of 5000 vertices with 3 initially infectious individuals. I examined the effects of different rewiring probabilities (0.0 to 0.5) and R_0 values (1.5 to 3.25) on epidemic measures. These measures included the total individuals who got sick, the length of the epidemic, the maximum number of infectious individuals at a time, and the day of peak infectious individuals).

Results

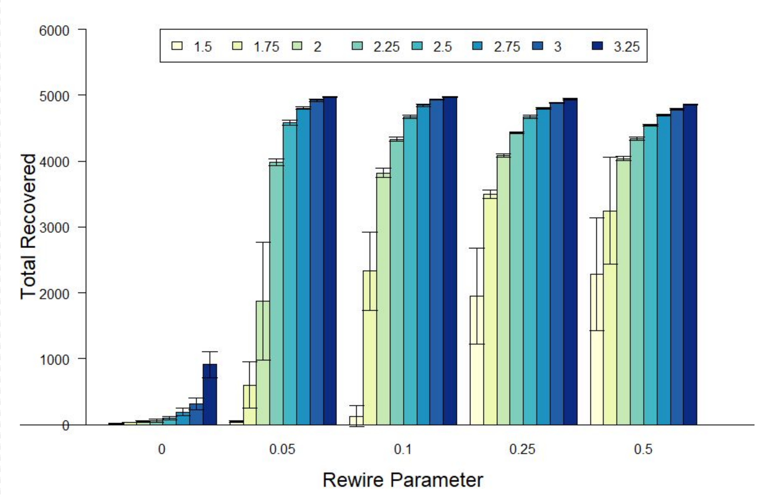

Figure 6. Total individuals who were infected throughout the epidemic at different rewiring parameters (x-axis) and R_0 values (legend).

The R_0 value and the rewiring probability both have a significant impact on the number of individuals who get sick in the population (R_0: df = 7, F = 408.78, p < .001; rewire.p: df = 4, F = 1351.32, p < .001). In addition, R_0 and rewire.p have a significant interaction with each other (df = 28, F = 38.46, p < .001). Figure 6 shows that the number of sick individuals increases dramatically at rewire.p = 0.05 and when R_0 exceeds 2.25. As the connectedness of the network increases (rewire.p increases), R_0 seems to have a lesser impact on the disease spreading through the population; a disease with R_0 = 1.5 that would not infect many in a system with fewer shortcut connections, seems to still take off in networks where rewire.p ≥.25.

The R_0 value and the rewiring probability both have significant impacts on the length of the epidemic and a significant interaction with each other (Figure 7, R_0: df = 7, F = 9.782, p < .001; rewire.p: df = 4, F = 26.502, p < .001; interaction: df = 28, F = 27.059, p < .001). Once again, we see a change in the interaction between R_0 and rewiring parameter after rewire.p = 0. When rewire.p = 0, increasing R_0 values dramatically lengthens the epidemic. As the network becomes more connected, increasing R_0 values causes the disease to run through the population more quickly.

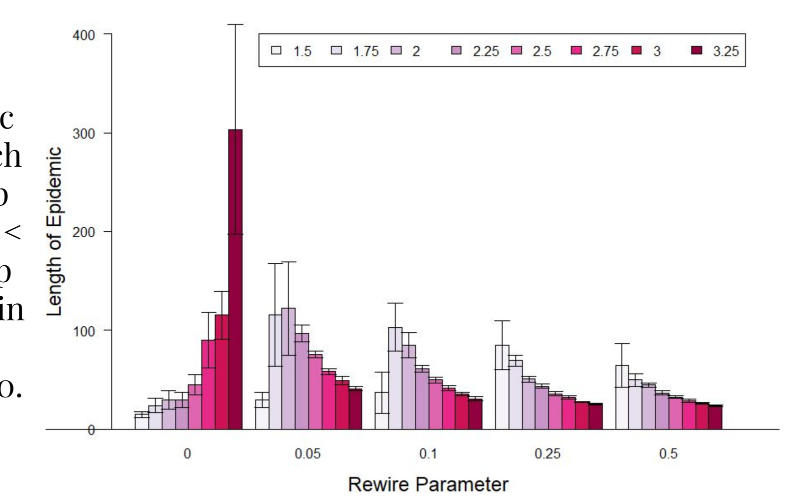

Figure 7. Overall length of the epidemic at different rewiring parameters (x-axis) and R_0 values (legend).

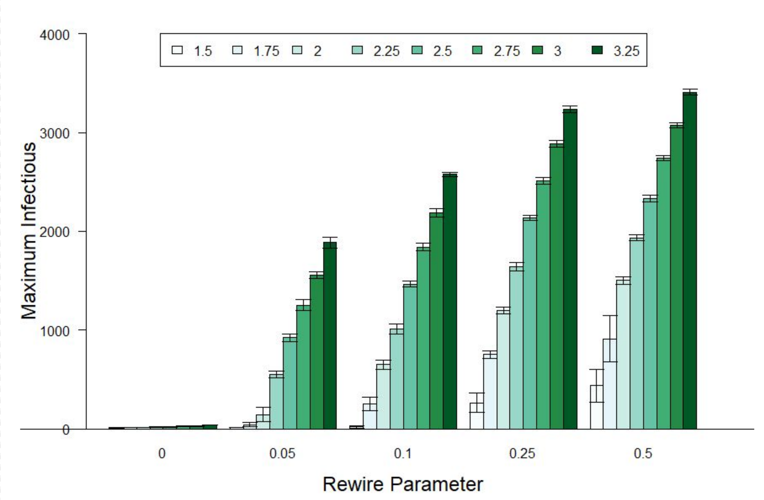

Figure 8. The maximum infectious individuals at one time at different rewiring parameters (x-axis) and R_0 values (legend).

Again, the R_0 value and rewiring probability have significant impacts on the maximum infectious individuals at a time and interact significantly with each other (Figure 8, R_0: df = 7, F = 4252.4, p < .001; rewire.p: df = 4, F = 8120.9, p < .001; interaction: df = 28, F = 298.5, p < .001). Here we see a gradual increase in the "height" of our infectious peak both as rewire.p increases and as R_0 increases. At rewire.p = 0, the maximum infectious is incredibly low regardless of the R_0 value. This makes sense when we see that the length of the epidemic under these circumstances was very high; the disease seemed to go through the population slowly and steadily.

As with the other measures, R_0 and rewire.p have a significant impact on when the apex occurs (Figure 9, R_0: df = 7, F = 11.242, p < .001; rewire.p: df = 4, F = 46.748, p < .001). They also have significant interaction with each other (df = 28, F = 5.073, p < .001). Again, there is a change in how R_0 impacts when the peak occurs at rewire.p = .05. Before then, the peak occurs relatively quickly regardless (although there is some increase with R_0). When rewire.p = .05 there is a turning point at R_0 = 2 when the peak is the farthest out. With higher connectedness, the peak gets closer with higher R_0 values.

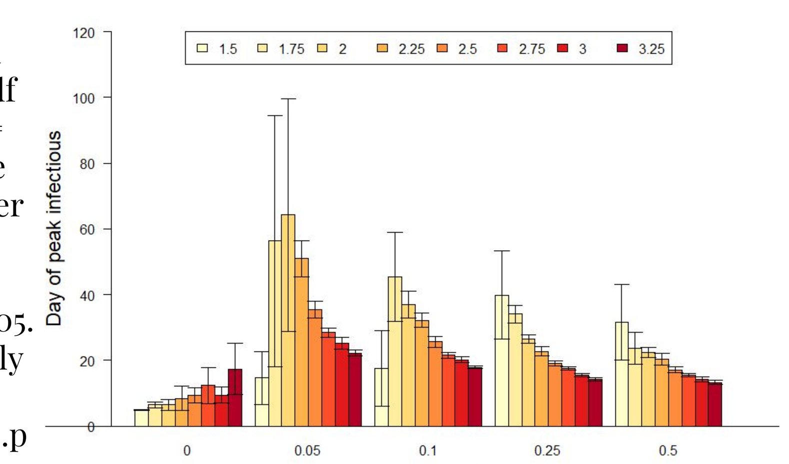

Figure 9. The day of peak infectious individuals at different rewiring parameters (x-axis) and R_0 values (legend).

Conclusion

R_0 values and the rewiring parameter have clear and significant impacts on epidemic measures. These parameters also have a significant interaction with each other. Throughout the epidemic measures, there looks to be a turning point at rewire.p =.05 where there is a change in how R_0 affects the circumstances. Situations for which the rewiring parameter is zero are uniquely mild. This is the situation that government and health officials are aiming towards now. Use of social distancing is similar to diminishing the rewiring parameter. Rewire.p = 0 would be ideal in our situation, by having people interact only with their immediate families/who they live with and no one else, but this is complicated by the need for essential services such as grocery stores as well as essential personnel going to their jobs. When the rewiring parameter is between .05 and .1, there is a turning point in the R_0 values around R_0 = 2 where past this value, epidemic measures get much worse: almost all of the population gets sick, the length of the epidemic becomes very short, and the day of peak infectious individuals comes much faster. This is an issue because it is likely to overwhelm hospitals; a shorter epidemic with a closer apex means less time to prepare with supplies for the surge of individuals requiring medical attention. We are readying ourselves for this issue with COVID-19 right now, with the hopes that social distancing will lengthen the epidemic long enough for healthcare systems to prepare. This turning point in R_0 is particularly interesting due to the newly estimated basic reproduction number of COVID-19. The Imperial College Group has assumed this strain of coronavirus to have an R_0 of 2.4, with a broader range between 2.0 and 2.6. Depending on the true value of COVID-19's basic reproduction number and society's attitude, social distancing could be extremely beneficial, or simply ineffective. If people do not adhere to the guidelines of social distancing (efforts are not obtaining a rewiring probability of very near to zero) then past R_0 of around 2.5 the resulting values for each epidemic measure become close in range to one another regardless of the connectedness of individuals. With COVID-19 so close to this critical R_0 value, it is important for everyone to practice social distancing and clean hygiene; if not, the efforts will be wasted.

References

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