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Technology in Teaching Mathematics: Desmos

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Erratum
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Abstract

This paper centers on research-supported ways of implementing dynamic geometry software into mathematics courses, specifically into high school geometry. The Next Generation Standards state, "For scaffolding purposes, the use of a variety of tools and methods for construction is encouraged. These scaffolds include compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc. Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena using visualization, reasoning, and geometric modeling to solve problems, in much the same way as computer algebra systems allow them to experiment with algebraic phenomena. Students can create geometric models and ideas to solve not only problems in mathematics, but in other disciplines or everyday situations." I discuss an overview of the research that supports the implementation of such mathematical technology to further student learning and achievement, as well as criteria for effective implementation of such technological activities. Moreover, this paper addresses the obstacles educators face when implementing technology, as well as suggestions to overcome said obstacles. Finally, I share a sample lesson plan (see Appendix B) using Desmos, a computer application created according to the research criteria that engages high school geometry students in exploration of transformations using dynamic geometry software.

Technology is constantly advancing and growing more and more prominent in our lives. Today's students are growing more fluent in technology. Schools have been trying to increase the incorporation of technology into classrooms for many reasons, motivation being one of them. Student achievement, attitude, and motivation are improved by proper incorporation of technology. Mathematics lends itself particularly well to the use of technological aids. Manipulatives are crucial to developing understanding in mathematics, as they allow for multimodal learning, which helps to build deeper conceptual understanding. Although not with handheld manipulatives, technological exploration is a useful way to explore mathematics. Technology gives the ability to look at multiple examples in a short period of time and manipulate the examples on the screen to draw conclusions similar to handheld manipulatives. The issue surrounding implementation is that often teachers do not fully understand the technology themselves. The purpose of this project is to discover how to properly implement technology into mathematics classrooms and evaluate the resources available to teachers. After extensive research, we have come up with a comprehensive list of criteria for picking mathematical technology to implement and incorporating it into lesson plans and created a sample lesson using Desmos that meets these criteria (see Appendix C).

Importance of Technology in the Classroom

School boards are making a push to implement technology more purposefully into classrooms. The New York Next Generation standards states,

> for scaffolding purposes, the use of a variety of tools and methods for construction is encouraged. These scaffolds include compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc. Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena using visualization, reasoning, and geometric modeling to solve problems, in much the same way as computer algebra systems allow them to experiment with algebraic phenomena. Students can create geometric models and ideas to solve not only problems in mathematics, but in other disciplines or everyday situations. (NYSED, 2019, p.125)

In other words, technology is essential to helping students look for mathematical patterns and develop problem solving skills. According to Sarama (2009), using computer tools, especially for geometry, makes students aware of the motions that are being performed (i.e. transformations). Moreover, the abstract nature of mathematics lends itself to the use of manipulatives so that we may develop concepts and understand said abstractions (Roschelle et al., 2010). Specifically, geometry concepts such as transformations are difficult to visualize without some form of representation, which cannot be given by textbooks (Adelabu et al., 2019). Since geometry is such a visual area, it naturally lends itself to the use of technology to create, view, and manipulate visual representations of the objects involved (Hollebrands & Dove, 2011). Moreover, Dynamic Geometry Software (DGS) provides new opportunities for students to engage in reasoning and sense-making activities (Hollebrands & Dove, 2011). When given time to explore with technology, students are guided to focus on the mathematical processes involved rather than the answers. The students are able to more clearly recognize patterns in the motions and develop a stronger conceptual understanding of the mathematical processes involved in the content being studied (Erlwanger, 2004). When exploring transformations with DGS, students can perform transformations with more accuracy, explore their properties, and consider what happens when two or more transformations are composed. Thus, students are discovering important properties on their own through investigation, which is essential to student learning (Hollebrands & Dove, 2011).

There are different types of knowledge: sensory-concrete, which is the need for sensory material to make sense of a concept or procedure, abstract, which is generalized and often symbolic knowledge, and finally integrated-concrete, which is a combination of the two (Sarama & Clements, 2009). Technology allows us to develop a combination of these types of knowledge, making the student's understanding of the material stronger. Moreover, computer manipulatives tend to be more user friendly because they are more manageable and precise for students. Additionally, computer manipulatives such as Desmos and GeoGebra provide immediate feedback, which allows students to monitor themselves and gauge their understanding as they go. Using such manipulatives allows students to replay, alter, and immediately reflect on a sequence of actions, and truly see how changes affect the transformation as a whole. The study conducted by Sarama (2009) showed that students with computer manipulatives outperformed their peers who were restricted to physical manipulatives or paper and pencil. Utilizing multiple modes of teaching, combining physical, technical and traditional methods will allow for more effective learning by the students (Roschelle et al., 2010). This ties back into the idea that fully integrated concrete knowledge is a stronger form of knowledge to have because it leads to deep conceptual understanding.

As Sarama's study showed, technology helps increase student achievement and attitude in the classroom. Technology helps learners and teachers in multiple aspects in the mathematics classroom, which allows permanent and effective learning (Adelabu et al., 2019). Using a different method of learning yields more permanent learning because it represents fully integrated concrete knowledge. The nature of educational technology such as Desmos and GeoGebra "helps teachers create learning contexts that were not previously possible with traditional teaching methods" (Eyyam & Yarata, 2014, p. 32). Technology opens doors for new paths of learning that facilitate creative and imaginative thinking. Students have an opportunity for individualized and flexible learning when technology is utilized in the classroom. Additionally, according to Eyyam's study, when technology is implemented properly and appropriately, it is guaranteed to have a positive impact on student achievement. In addition to positively impacting student achievement, technology has been shown to positively impact student attitude. Technology enhances students' motivation and helps make learning more engaging. It is important that students are familiar with technology before it is implemented into lessons, as this will make the lesson more enjoyable for the students (Eyyam & Yarata, 2014).

The Case for Dynamic Geometry Software

Often, teachers will try to fill the technology requirement with calculators, however research has shown that in order to truly benefit students, geometry teachers should incorporate Dynamic Geometry Software (DGS) into their classrooms. Adelabu and colleagues define Dynamic Geometry Environments as, "particular technology tools that have been used in the teaching and learning of geometry to assist learners in moving beyond the specifics of a single drawing to generalizations across figures and shapes" (2019, p. 54). The defining characteristics of DGS programs include a set of primitive objects, the ability to construct parallel and perpendicular lines, tools for performing transformations and calculations, ability to drag, measure, animate and hide objects, and finally an ability to create procedures (Hollebrands & Dove, 2011). Based on these criteria, Desmos can be defined as a form of DGS. GeoGebra is one of the most popular forms of DGS, partially because it is a free resource, and research has shown that it is more effective at enhancing the learner's performance and understanding than traditional pencil and paper (Adelabu et al., 2019).

As stated previously, technology positively impacts learners' achievement and attitude in the classroom. Specifically, DGS is important to incorporate in the classroom because it serves as an object of education, which affects learning objectives and serves as a medium to improve the teaching and learning process (Arbain & Shukor, 2015). They found that students enjoyed using DGS, especially when they were familiar with the technology being used: "Students in the 21st century are computer-literate and the opportunities to learn using technology support will attract major attention" (Arbain & Shukor, 2015, p. 212). Technology, when implemented properly, can significantly improve students' engagement and learning in the classroom.

The DGS primarily studied in this research was GeoGebra. GeoGebra is an action technology that provides new mathematical tools and scenarios that engage students in meaningful mathematical activity and problem solving (NCTM). Although I used Desmos to create the activity, most research available exists on GeoGebra. Desmos is essentially the same as GeoGebra, except instead of the activity being a worksheet, it is presented in slide form. With Desmos, the students can progress through the slides at their own pace in order to complete the activity and learn at a rate proper for them. The benefits of both GeoGebra and Desmos are that these websites are free, so schools can use them without having to purchase subscriptions, and students can access them outside of school. Due to the easy-to-use nature of GeoGebra, there is an online community surrounding it, which allows educators to share ideas with ease. "The use of GeoGebra encouraged the [students] to take a more active independent role in their learning by answering a series of questions…fosters experimental, problem oriented, independent learning, and discovery learning of mathematics" (Getenet, 2018, p. 223). When interactive technology is incorporated into classrooms, research shows that students begin to take control of their learning.

Criteria for Implementing Technology

One of the key components to incorporating technology into the classroom is making sure that it is done effectively. Multiple studies presented criteria for effectively implementing technology. We will discuss the different guidelines presented and create a compilation of criteria that should be considered while creating technology-based lessons. Additionally, technology is versatile. It can intensify instruction, covering the basics of the material as well as more advanced topics all depending on the mode of implementation (Roschelle et al., 2010).

Beginning with the National Council of Teachers of Mathematics (NCTM) guidelines, when planning, one must first identify the learning objectives for the lesson. After this is done, a task must be created or modified that provides the students with opportunities to meet the objective and engage in reasoning and sense making to support learning as they progress through the task. Once the task is chosen, one must decide how to implement the lesson, this includes organization of students, presentation of the task, and time allotted to the task. At the end of the lesson, the teacher should manage class discussion regarding students' observations and conjectures and help them draw significant connections that they may have missed (NCTM).

Getenet recommends another checklist on how to evaluate the implementation of technology. This checklist is based on seven categories of the TPACK (Technological, Pedagogical, and Content Knowledge) model. First is knowledge needed to use a particular technology. This includes ensuring students have the skills necessary to properly use the technology, as well as ensuring that the knowledge and skills gained from the technological activity can be transferred to new situations. In other words, when implementing technology, teachers must make sure that they themselves and the students can use said technology, and that the proper supports are implemented throughout the lesson so that students can effectively learn from the activity. Next the teacher must have all students' learning styles in mind, i.e., knowledge about process and practices of teaching. Teachers should be able to incorporate real-world issues and solve authentic problems using teaching resources, address diverse needs of all learners by using learner-centered strategies, and provide equitable access to appropriate resources. A more self-explanatory criteria is that teachers must demonstrate knowledge of the subject matter to be taught. The teacher must have a solid understanding of the core principles and procedures involved with the subject matter in order to ensure that the technology being implemented in the lesson is appropriate to the overall learning goals. Thus, the teacher should include a clear introduction to the topic and learning goals, have sufficient knowledge of the lesson, demonstrate confidence in mathematical concepts related to the lesson, and use appropriate materials in relation to the lesson being taught. The teacher must next ensure that they have knowledge of how pedagogical approaches are suited to teaching the content. That is, the teacher must possess the ability to integrate teaching approaches that arouse creativity and apply teaching approaches, which give more authority to students in solving mathematics problems. Next on the checklist is reflecting on how technology and content interact in effective teaching. The teacher must understand how the subject matter can be changed by use of technology. There is a clear link between technological and mathematical knowledge. Teachers should design relevant learning experiences that incorporate technology used to promote learning and introduce fundamental concepts through exploration. Again, teachers must have knowledge of how to use the technology with different pedagogical approaches. Last, teachers must have knowledge that is more than the sum of the other three components. To make a proper choice of technology in relation to mathematics concept and pedagogy, teachers should integrate the components of the TPACK model in order to promote creative thinking and apply TPACK to promote reflection and to clarify conceptual thinking. This full checklist can be found in Appendix A (Getenet, 2018).

The last evaluation method to be discussed is Kovacs' method for evaluation online educational materials. This is based on four general principles, from which he derives sixteen quality identifiers. The first principle is the *multimedia principle*. The educational activity should utilize both words and graphics rather than just words. Next is the *contiguity principle*, which requires that corresponding words and graphics are placed near each other on the page. The *coherence principle* requires an element of simplicity. Although it is tempting to add interesting material such as detailed textual descriptions, entertaining stories or background music, this can ultimately hurt the students' learning and should be avoided. Finally, we have the *personalization principle*, which states that the activity should address the students directly using a conversational style. These four general principles yield the three groups containing the sixteen quality identifiers. The first group is *layout*. The layout of the activity should avoid scrolling, have a brief explanation, require few tasks, and avoid distractions. The next group is *dynamic figures*. This group names a quality activity one that is interactive, easy-to-use, appropriate size, has dynamic text placed close to corresponding objects, avoids the clutter of static text, and is readable upon first appearance (is not overwhelming). Finally, the last group is *explanations and tasks*. All explanations and tasks should have a small number of specific questions that refer to the applet and vocabulary reflecting that the audience is the learners, as well as tasks, which include demonstration figures with no additional tasks or questions attached (Kovacs & Cole, 2019).

A comprehensive guideline blends these three guidelines (NCTM, TPACK, and Kovacs' method) together and includes mathematical viewpoint, pedagogical viewpoint, layout, dynamic elements, and instruction and text. It is important to keep everything that the NCTM, Getenet, and Kovacs discussed in mind when creating and selecting technological activities for the classroom. Doing so will ensure that the activity fits properly into the lesson, is used effectively, and is conducive to students' achievement of the lesson objectives. Additionally, there are some key steps to success when implementing technology. When learners have prior knowledge of how to use technology such as GeoGebra, the full potential of the discovery approach may be effectively realized (Getenet, 2018). Thus, the first step to successfully integrating a technology-based activity is having students familiarize themselves with the program. Next, one should go through the various check-lists and ensure that the technology will aid students in achieving the objectives for the lesson, that the activity satisfies the TPACK model, and that the activity is built effectively so that it will not be distracting to the students. Additionally, effective technology will "aid students in building, strengthening, and connecting various representations of mathematical ideas" (Sarama & Clements, 2009, p. 146).

As discussed in detail previously, DGS lends itself nicely to exploration. Additionally, DGS can be used for the pedagogical strategy called delayed scaffolding, in which teachers do not immediately provide scaffolding to students. Instead, the students work independently while the teacher circulates the room. The teacher can ask guiding questions to lead students in the right direction. With this strategy, students can use the technology to explore what they have already learned in order to discover a deeper meaning. This is a metacognitive strategy because it puts students in charge of their own thinking and encourages them to become aware of their own learning.

Obstacles and Solutions

Since we have discussed effective ways to incorporate technology into a lesson, we now provide warnings about possible constraints or obstacles to achieving this. One more obvious obstacle is a lack of necessary equipment. Semenikhina suggests the BYOD (bring your own device) approach. With this, students can bring in their own cloud-based devices. Since GeoGebra (and Desmos) are accessible on any device, if only a handful of computers or iPads are available to the class, students may use their phones to complete the selected activity. Although this is not ideal, it is a solution for schools that cannot provide a personal laptop for each student, or for when a teacher simply cannot get access to a computer lab. Another obstacle is a lack of training for teachers (Eyyam & Yaratan, 2014). It is important to train teachers in these new technologies so that they can expertly and effectively incorporate technology into their classrooms (Kovacs, 2019). Lifelong learning and professional development can solve this problem. Providing teachers with the necessary information, aids, equipment and opportunities for professional development will ensure that they master the technology (Eyyam & Yaratan, 2014). Additionally, teachers should be fully aware and involved in the integration process to prevent problems. To avoid obstacles to implementing technology, teachers should practice "good teaching," which requires "an understanding of the representation of mathematical concepts using technologies; pedagogical techniques that use technologies in constructive ways to teach content; knowledge of what makes concepts difficult or easy to learn, and how technology can help reformulate some of the problems that students face" (Kovacs, 2019, p. 137). Teachers must think through the application of the activity to the topic, why is the technology being used and how will it advance student learning. Finally, teachers must be able to identify the cause of student errors, since errors may simply be due to lack of understanding the technology rather than lack of understanding a concept (Kovacs, 2019). Additionally, students should be very familiar with technology as well, as this will help prevent errors in their work.

The Lesson and Activity

My research project culminated in an example lesson plan that incorporates DGS. While technology can be useful for all subjects of mathematics, for the purposes of creating a lesson, the topic I chose to focus on is transformation geometry. This lesson is an exploratory introduction to translations. Before considering an activity, I formulated the following objective for my lesson: Given Chromebooks, previous instruction on how to work Desmos, and an introduction to translations, students will explore this transformation as a function on geometric figures in order to complete an online task on Desmos which demonstrates how this transformation works and its various properties. The lesson focuses specifically on translations, so the key properties are preservation of orientation and size, slide by a vector (specific line length and slope), and that image segments are congruent and parallel (to each other and to the vector of translation). Once the objective was established, I began to create an activity on Desmos. Desmos was used instead of GeoGebra because it is also a free form of DGS, however it provides the teacher and student with immediate feedback, which is beneficial for a classroom. Additionally, the teacher can pause the slides, in order to emphasize a property for students if needed or simply grab the students' attention. Moreover, the data from the students' work is stored on Desmos, so the teacher may go back and see students' results later to evaluate their understanding.

This exploration-based activity is linked in the lesson plan under the "Materials" section and can also be found in Appendix B. There are opportunities for students to explore, make conjectures, explore more and adjust their conjectures and so on. The lesson design allows students to develop a deep understanding of the inner workings of translations. There are also slides that test how well students understood what they were doing, by asking them to fix a mistranslated object by adjusting the image to the appropriate location. Additionally, there is a question asking students to determine the vector of translation when given the preimage and image. Students can use their understanding of the image segments, as built during the beginning of the activity, to apply and advance their understanding to a different scenario in order to solve the given problems. These two questions also embody the problem solving that exists within the lesson. This activity allows for productive struggle with the material, which in turn will help students develop a stronger understanding of the material.

I created this Desmos activity for the lesson according to the guidelines discussed in this paper. The activity is to be implemented such that students can explore, but are given the appropriate support when necessary, and there is time built in after the activity for a class-wide discussion in which the students can verbalize their conjectures in a precise manner, and critique each other's ideas. Additionally, during this whole class discussion, the teacher can prompt students with questions to help draw out connections they might have missed during the activity. For instance, if a student did not recognize that the image segments are congruent and parallel to each other and the vector of translation, the teacher can go back to a slide and ask more directed questions focusing on this missed connection. During this whole class discussion, students can also raise any concerns or ask any unanswered question they may have from the activity itself.

Conclusion

Overall, this research helped me to better understand the importance of using technology in mathematics classrooms. Technology, when incorporated effectively, is proven to have a positive impact on student motivation, attitude, and achievement.

Dynamic geometry software is especially beneficial to student learning and should be incorporated where applicable. Additionally, I learned the criteria to use to evaluate and effectively incorporate such technology into a lesson (NCTM, Kovacs, TPACK). Keeping this research in mind will allow you to create a worthwhile mathematical task whose impact may be enhanced through effective incorporation of technology.

Appendices

All appendices are located in the online edition of *Proceedings of GREAT Day 2020*, found at https://knightscholar.geneseo.edu/proceedings-of-great-day/vol2020/iss1/8/

Appendix A: TPACK Guideline

This Appendix contains the TPACK Guideline for implementing technology into the classroom. The guideline provides a checklist for educators to consider when creating their lesson plans that incorporate technology. This checklist will allow educators to determine if the technology they are implementing is appropriate, as well as ensuring that it is incorporated in a way that promotes creative thinking, reflection, and clarify conceptual thinking.

Appendix B: The Lesson

This Appendix has a sample lesson plan which demonstrates how to integrate technology into the classroom. In this lesson plan, a sample anticipatory set is provided to lead students into the exploration of translations. Before implementing the activity, the teacher should set clear expectations, but after that the work is left up to the students. In the last few minutes of class, it is important to bring the class together for a debriefing of what knowledge they gained from the activity. The teacher should hold a discussion that is led by the students and their discoveries, and simply prompt responses when necessary. In this Appendix, there is also a sample homework assignment that can be given to the class as a follow up activity. The answer key is also contained here.

Appendix C: The Desmos Activity

This Appendix contains the slides from the Desmos Activity (which is also linked). There are screen caps of each slide that are contained in the activity. In order to work with the activity and implement it, follow the link that is in this Appendix. The activity has students make conjectures and scaffolds students understanding of translations on and off of the coordinate plane. By the end of the activity, students are able to construct the vectors of translation and translate polygons.

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